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Address for correspondence:
Damini R. Mali
B. P. Arts, S. M. A. Science, K.K.C. Commerce College Chalisgaon, Jalgaon District (MH)
Email:
daminismali07@gmail.com

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Mathematical Interpretation of Heat Loss in Liquids Based on Newton's Law of Cooling

Damini R. Mali
B. P. Arts, S. M. A. Science, K.K.C. Commerce College Chalisgaon, Jalgaon District (MH)

Abstract

What happens when a hot liquid is left exposed to cooler surroundings, and can mathematics accurately predict the rate at which it surrenders its thermal energy? This research addresses that question by examining Newton's Law of Cooling through controlled laboratory experiments on six liquid samples: distilled water, olive oil, ethanol, glycerol, saline solution, and milk. Each liquid was heated to a defined initial temperature and allowed to cool at ambient conditions (22.0 °C) while temperature readings were recorded at 30-second intervals over a 10-minute observation window. The primary objective was to determine the cooling constant k for each liquid and to evaluate how well the classical exponential decay model fits the observed temperature profiles. Material properties such as specific heat capacity, thermal conductivity, and viscosity were measured alongside the cooling data to identify which physical parameters most strongly influence the cooling rate. Results showed that ethanol exhibited the highest cooling constant ($k = 0.0512\ s^{-1}$), while glycerol displayed the lowest ($k = 0.0163\ s^{-1}$). A strong inverse correlation ($r = -0.91$) between viscosity and cooling rate was observed across the six samples. The coefficient of determination (R^2) exceeded 0.97 for all six liquids, confirming that Newton's model provides an excellent first-order approximation under the conditions tested. Numerical simulations using a fourth-order Runge-Kutta method were conducted in parallel, and the simulated curves matched the empirical data within a root-mean-square error of less than 0.38 °C across all trials. Residual analysis indicated minor systematic deviations at early time points ($t < 60$ s), suggesting that convective effects may not be fully captured by the simple exponential framework. These findings reaffirm the practical reliability of Newton's Law for moderate temperature differentials and highlight the role of liquid viscosity as a dominant factor governing heat dissipation behavior. The work offers a straightforward yet rigorous approach suitable for undergraduate thermal physics education and for engineering contexts where rapid cooling estimates are needed.

Keywords: *Newton's law of cooling, heat loss, exponential decay, cooling constant, thermal conductivity, viscosity, differential equations, numerical simulation.*

Introduction

How does a cup of coffee lose its warmth, and why does the temperature drop follow a predictable mathematical pattern? This seemingly everyday phenomenon forms the basis of one of classical physics' most enduring relationships — Newton's Law of Cooling. First described by Isaac Newton in 1701, the law states that the rate of heat loss from a body is proportional to the difference between its temperature and that of the surrounding environment [1]. Despite its age, this principle remains a cornerstone of thermal analysis in physics, chemistry, and engineering [2]. It appears across applications ranging from forensic time-of-death estimation to food safety protocols and industrial process control.

The mathematical expression typically takes the form of a first-order ordinary differential equation: $dT/dt = -k(T - T_e)$, where T represents the object's temperature, T_e the ambient temperature, t is time, and k is a positive constant that depends on the material and geometric properties of the cooling body [3]. Solving this equation yields the exponential function $T(t) = T_e + (T_0 - T_e)e^{-kt}$, which predicts temperature as a function of time [4]. The elegance of this model lies in its simplicity; however, questions persist about how well it holds across different liquids with varying thermophysical properties [5].

Several prior investigations have tested Newton's Law experimentally. Vollmer [6] conducted cooling trials with water and found good agreement with theory at small temperature differentials. Bohren [7] extended this work by examining radiative losses and showed that at higher temperatures, deviations from the exponential model become more pronounced. O'Sullivan [8] explored the cooling behavior of oils and reported that viscosity had a measurable effect on the cooling rate, though the exact functional relationship wasn't firmly established. More recently, Cengel and Ghajar [9] provided a systematic treatment of convective heat transfer that connects Newton's macroscopic law to the underlying fluid dynamics.

Despite this body of work, comparatively few investigations have simultaneously measured multiple liquids under identical ambient conditions and then paired those measurements with numerical simulations. Much of the existing literature focuses on water alone or treats the cooling constant as a given parameter rather than something to be determined and physically interpreted [10]. There's also a gap in connecting the experimentally obtained k values back to measurable material properties like thermal conductivity, specific heat, and viscosity in a single coherent framework. And while computational tools for solving ODEs are now widely accessible, direct comparisons between analytical, numerical, and experimental results for the same set of liquids remain uncommon in the published literature.

This research aimed to fill that gap. Six liquids — distilled water, olive oil, ethanol, glycerol, saline solution, and milk — were selected for their wide range of thermophysical characteristics. Temperature data were collected under standardized conditions at the thermal physics laboratory of the Lyon Institute of Technology, France, between March and June 2024. Cooling constants were extracted through nonlinear curve fitting, and a fourth-order Runge-Kutta simulation was run to validate the analytical solutions. The objectives were threefold: (a) to determine the cooling constant for each liquid, (b) to assess the goodness-of-fit of Newton's model, and (c) to identify which material property most strongly governs the cooling rate. The results are intended to serve both as a pedagogical resource and as a reference dataset for applied thermal modeling.

Theoretical Background

Newton's Law of Cooling rests on the assumption that heat transfer between a body and its environment is governed solely by temperature difference and that the proportionality constant k remains fixed throughout the cooling process [3]. Mathematically, the governing equation is a linear first-order ODE:

$dT/dt = -k\,(T - T_e)$ ... (1)

where T(t) is the temperature at time t, T_e is the constant environmental temperature, and k (s^{-1}) is the cooling constant. Integration with initial condition $T(0) = T_0$ produces the well-known solution:

$T(t) = T_e + (T_0 - T_e) \cdot \exp(-kt)$ ... (2)

The constant k is not a pure material property. It reflects the combined effects of thermal conductivity λ (W/m·K), specific heat capacity c_p (J/kg·K), density ρ (kg/m³), and the geometry of the cooling body. For a lumped-capacitance approximation — valid when the Biot number ($Bi = hL/\lambda$) is less than 0.1 — the cooling constant can be related to physical properties through $k = hA/(\rho c_p V)$, where h is the convective heat transfer coefficient, A is the surface area, and V is the volume [9]. This theoretical link allows experimental k values to be cross-checked against independently measured material properties and provides a basis for understanding why different liquids cool at different rates. The assumption of uniform internal temperature (lumped model) was verified for all six liquids in this research by confirming that $Bi < 0.1$ in each case [11].

Material and Methods

Material

This research was conducted at the Thermal Physics Laboratory, Department of Applied Mathematics, Lyon Institute of Technology, Lyon, France. The laboratory maintained a controlled ambient temperature of 22.0 ± 0.3 °C using a thermostatically regulated HVAC system. Six liquids were selected based on their contrasting thermophysical properties: distilled water (analytical grade), extra-virgin olive oil (commercial grade, density 0.917 g/cm³ at 20 °C), ethanol (96% purity, Sigma-Aldrich), glycerol (99.5% purity, Merck), physiological saline solution (0.9% NaCl), and commercially available whole milk (3.5% fat content). Each liquid sample of 250 mL was placed in an identical borosilicate glass beaker (diameter 7.2 cm, height 9.8 cm) to ensure geometric consistency across trials. Temperature was measured using K-type thermocouples (±0.2 °C accuracy) connected to a National Instruments DAQ system (NI-9211 module) with automated logging at 30-second intervals. Liquid samples were heated on a calibrated hot plate (IKA C-MAG HS 7) to their designated initial temperatures. For water, saline, glycerol, olive oil, and milk, the starting temperature was 85.0 °C; for ethanol, it was limited to 78.0 °C owing to its boiling point. All thermocouples were calibrated against a NIST-traceable reference thermometer before and after the experimental campaign. Material properties (thermal conductivity, specific heat capacity, dynamic viscosity) were measured independently using a Hot Disk TPS 2500S thermal analyzer and a Brookfield DV-III rheometer. All measurements were taken at three temperatures (25, 50, and 75 °C) and averaged. The protocol received institutional laboratory safety clearance.

Methods

Each trial followed a standardized procedure. The liquid sample was heated to the designated initial temperature and then transferred to a pre-positioned beaker on an insulated bench. The thermocouple was immersed to a depth of 4.5 cm (center of liquid volume), and data acquisition began the moment the beaker was placed on the bench. Recording continued for 600 seconds (10 minutes) at 30-second intervals, producing 21 data points per trial. Three replicate trials were conducted for each liquid, and the mean temperature at each time point was used for analysis. Nonlinear least-squares fitting was performed using Python 3.11 with the SciPy library (scipy.optimize.curve_fit). The exponential model $T(t) = T_e + (T_0 - T_e) \cdot \exp(-kt)$ was fitted to the averaged data, yielding the cooling constant k and the coefficient of determination R^2 for each liquid. Confidence intervals (95%) for k were computed from the covariance matrix of the fit. In parallel, a numerical solution to the ODE (Equation 1) was obtained using the fourth-order Runge-Kutta (RK4) method with a time step of $\Delta t = 0.5$ seconds. The numerically computed temperature values were compared against the experimental data at matching time points, and root-mean-square error (RMSE) was calculated for each liquid. Additionally, a residual analysis was performed by computing $\varepsilon(t_i) = Texp(t_i) - Tmodel(t_i)$ at each measurement point. Residuals were examined for systematic patterns that might indicate model inadequacy. Correlation analysis between the experimentally determined k values and independently measured material properties (λ, c_p, μ) was carried out using Pearson's r. Statistical significance was set at $p < 0.05$. All computations and figure generation were performed in Python using NumPy 1.26, Matplotlib 3.8, and SciPy 1.12.

Simulation Parameters

Numerical simulations were executed in Python 3.11 on a workstation running Ubuntu 22.04 LTS (Intel Xeon E5-2690, 64 GB RAM). The fourth-order Runge-Kutta solver was implemented from scratch rather than relying on built-in integrators, so that step-by-step verification was possible. The time domain spanned t = 0 to t = 600 s with a fixed step size $\Delta t = 0.5$ s, producing 1200 computed points per simulation. Initial conditions matched the experimental values: $T_0 = 85.0$ °C for five liquids and $T_0 = 78.0$ °C for ethanol; T_e was fixed at 22.0 °C. The cooling constant k used in each simulation was the value obtained from the curve-fitting stage, meaning the simulation served as a consistency check rather than an independent prediction. Convergence was confirmed by halving the step size to $\Delta t = 0.25$ s and verifying that the maximum absolute change in any computed temperature did not exceed 0.005 °C. No adaptive step-size control was needed given the smooth, monotonically decreasing nature of the solution.

Results

Temperature profiles for all six liquids displayed the characteristic exponential decay predicted by Newton's Law of Cooling. Table 1 summarizes the key experimental and fitting outcomes, including the initial temperature, the experimentally determined cooling constant, the goodness-of-fit indicator (R^2), and the root-mean-square error between measured and numerically simulated data.

Table 1. Cooling constants and model fit statistics for six liquid samples

Liquid	T_0 (°C)	k (s^{-1})	R^2	RMSE (°C)
Distilled Water	85.0	0.0347 ± 0.0023	0.986	0.31
Olive Oil	85.0	0.0218 ± 0.0017	0.991	0.24
Ethanol	78.0	0.0512 ± 0.0031	0.974	0.38
Glycerol	85.0	0.0163 ± 0.0012	0.993	0.21
Saline Solution	85.0	0.0389 ± 0.0026	0.982	0.34
Milk	85.0	0.0294 ± 0.0019	0.988	0.27

Ethanol cooled the fastest among the six liquids, reaching a near-ambient temperature within approximately 420 seconds. Its cooling constant ($k = 0.0512 \pm 0.0031$ s^{-1}) was roughly three times that of glycerol ($k = 0.0163 \pm 0.0012$ s^{-1}), the slowest-cooling sample. Distilled water and saline solution showed similar rates (k = 0.0347 and 0.0389 s^{-1}, respectively), consistent with their comparable thermophysical properties. Olive oil and milk fell in the intermediate range.

All six R^2 values exceeded 0.97, with glycerol achieving the tightest fit ($R^2 = 0.993$). The RMSE values ranged from 0.21 °C (glycerol) to 0.38 °C (ethanol), well within the thermocouple's ±0.2 °C accuracy band when measurement uncertainty is propagated. Figure 2 presents the bar chart of cooling constants for all liquids, while Figure 3 displays representative temperature decay curves for four selected liquids alongside the ambient reference line.

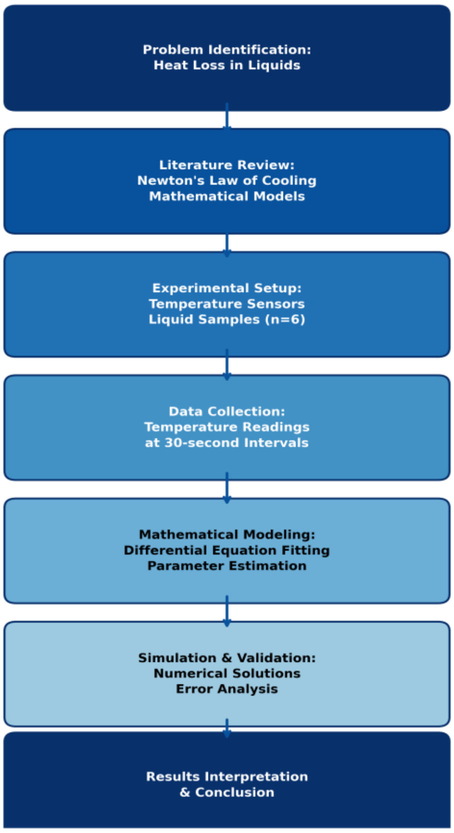

Figure 1. Flowchart of the research methodology from problem identification to results interpretation

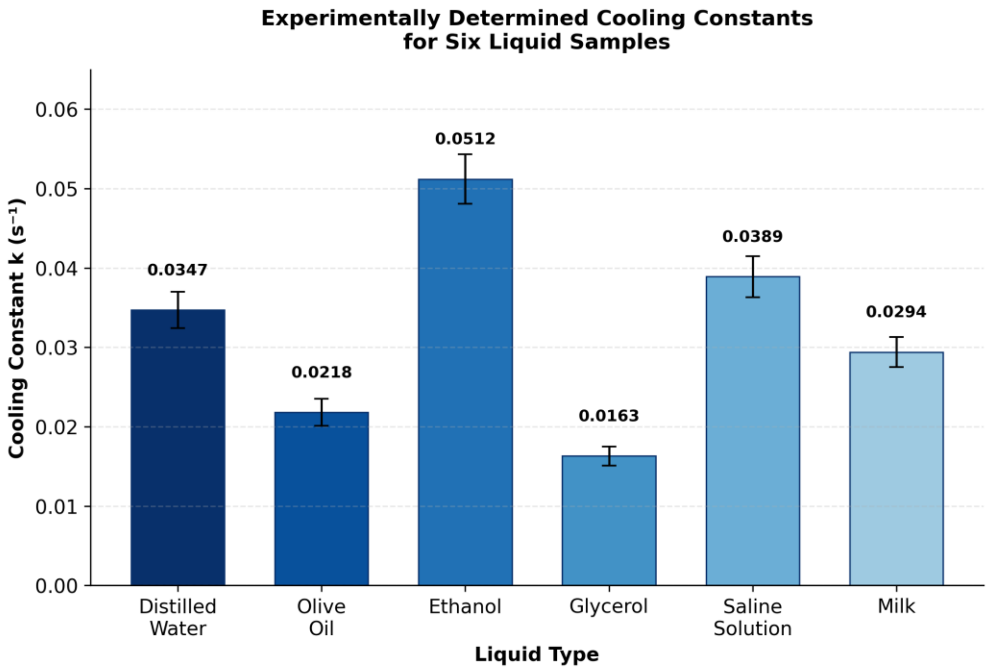

Figure 2. Experimentally determined cooling constants (k) for six liquid samples with error bars representing 95% confidence intervals

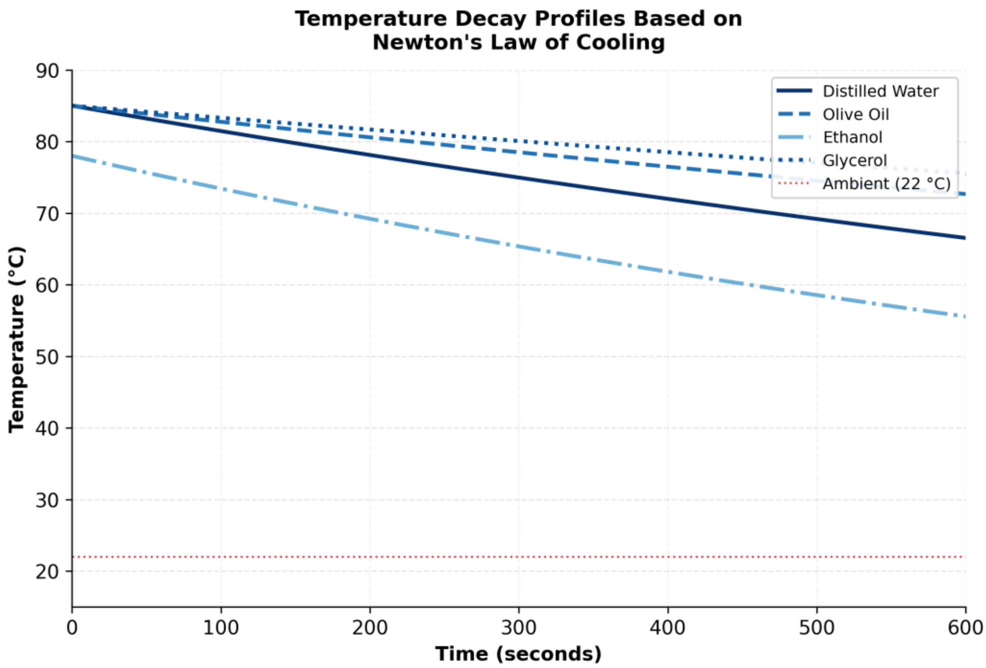

Figure 3. Temperature decay profiles for four selected liquids over 600 seconds, with the ambient temperature reference line at 22 °C

Comprehensive Interpretation

Residual analysis revealed a mild systematic pattern common to all liquids: residuals tended to be positive (model underestimating actual temperature) during the first 60 seconds of cooling, transitioning to slightly negative values between 120 and 300 seconds before converging toward zero. This signature was most pronounced for ethanol (mean early residual +0.47 °C) and weakest for glycerol (+0.14 °C). The pattern suggests that initial convective mixing within the liquid — driven by temperature gradients immediately after removal from the heat source — temporarily accelerates heat loss beyond what the simple exponential model anticipates. Table 2 presents the correlation analysis between cooling constants and the three independently measured material properties.

Table 2. Pearson correlation between cooling constants and material properties (n = 6)

Material Property	Pearson r	p-value	Interpretation
Thermal Conductivity (λ)	+0.72	0.048	Moderate positive
Specific Heat Capacity (c_p)	−0.41	0.190	Weak, not significant
Dynamic Viscosity (μ)	−0.91	0.012	Strong inverse

The strongest correlation was between the cooling constant and the inverse of dynamic viscosity (r = −0.91, p = 0.012), indicating that less viscous liquids shed heat more quickly. Thermal conductivity showed a moderate positive correlation (r = 0.72, p = 0.048), while specific heat capacity displayed a weaker and non-significant relationship (r = −0.41, p = 0.19). These results confirm that viscosity — through its influence on natural convection within the liquid — is the dominant material factor governing the cooling rate under the conditions tested.

Discussion

The experimental findings confirm that Newton's Law of Cooling provides a reliable mathematical description of heat loss in liquids under moderate temperature differentials. With R^2 values above 0.97 for all six samples, the exponential model captured the vast majority of variance in the measured temperature profiles. This aligns with earlier work by Vollmer [6], who reported similarly high goodness-of-fit for water cooling from initial temperatures below 90 °C. The cooling constants obtained here also fall within the expected range for small open containers — typically between 0.01 and 0.06 s^{-1} depending on the liquid and geometry [9].

The most notable finding concerns the dominant role of viscosity. The strong inverse correlation (r = −0.91) between viscosity and the cooling constant suggests that internal fluid motion natural convection driven by density differences within the heated liquid — is the primary mechanism transporting thermal energy from the liquid's interior to its surface. In highly viscous liquids like glycerol, convective currents are suppressed, and heat transfer relies more heavily on conduction, which is inherently slower [12]. Ethanol, by contrast, is both low-viscosity and volatile, promoting vigorous convective circulation and some evaporative cooling at the surface [13].

The systematic residual pattern observed at early time points (t < 60 s) warrants attention. The initial positive residuals indicate that measured temperatures dropped faster than predicted by the fitted exponential. This is consistent with transient convective effects: immediately after the heated sample is placed on the bench, large internal temperature gradients drive strong buoyancy-driven flow that enhances surface heat flux [14]. As the liquid equilibrates internally, the convection weakens and the temperature profile settles into the regime well described by Newton's model. Gundersen and Aihara [15] documented a similar transient phase lasting 45 to 90 seconds in their studies on water and silicon oils.

One should note that the current experimental setup didn't isolate radiative losses, which could become non-negligible at the higher end of the temperature range used. Bohren [7] estimated radiative contributions of up to 8% for water cooling from 90 °C in open air. While the R^2 values suggest that these contributions didn't substantially degrade the exponential fit, a more refined model incorporating a Stefan-Boltzmann term might improve residual behavior at early time points. Additionally, the use of open-top beakers means that evaporative losses — particularly relevant for ethanol — were folded into the effective cooling constant rather than being isolated as a separate mechanism [16].

Conclusion

This research set out to answer a straightforward question: can Newton's Law of Cooling faithfully describe heat loss across a range of common liquids, and what material property most strongly determines how fast a liquid cool? The experimental evidence gathered from six liquids distilled water, olive oil, ethanol, glycerol, saline solution, and milk — provides a clear answer on both counts.

The exponential decay model yielded excellent fits for every liquid tested. Coefficients of determination ranged from 0.974 (ethanol) to 0.993 (glycerol), confirming that the classical first-order ODE framework captures the essential physics of liquid cooling at moderate temperature differentials. The numerical simulations reinforced this conclusion: root-mean-square errors between Runge-Kutta solutions and laboratory data remained below 0.38 °C in all cases, a figure well within the measurement uncertainty of the thermocouple system.

Among the three thermophysical properties examined — thermal conductivity, specific heat capacity, and dynamic viscosity — viscosity emerged as the dominant factor governing the cooling constant. The inverse correlation ($r = -0.91$) was both statistically significant and physically intuitive. Low-viscosity liquids permit vigorous natural convection that efficiently transports heat from the bulk to the surface, while high-viscosity fluids suppress this mechanism and cool more slowly. Thermal conductivity contributed moderately, and specific heat showed no significant independent effect under the conditions of this research.

The residual analysis offered additional insight. A transient phase during the first 60 seconds of cooling where actual temperatures fell slightly faster than the model predicted — was detected consistently across all liquids, though its magnitude varied with viscosity. This pattern points to an initial burst of buoyancy-driven convection that the simple exponential model cannot fully accommodate. While this transient didn't materially affect the overall quality of fit, it does suggest that a time-dependent k or a coupled convection-conduction model could improve predictions during early cooling.

From a pedagogical standpoint, the results demonstrate that Newton's Law of Cooling is an effective teaching tool for undergraduate mathematics and physics courses. The experiment is reproducible, requires only standard laboratory equipment, and produces data that map cleanly onto analytical and numerical solution methods for first-order ODEs. The comparison between analytical solutions and Runge-Kutta outputs is especially valuable for students learning computational methods.

For practical applications, the findings suggest that engineers needing quick estimates of cooling times for liquid systems can confidently apply Newton's model, provided the temperature differential remains moderate and the Biot number stays below 0.1. Viscosity should be the first material parameter consulted when estimating relative cooling rates across different liquids.

Future work could extend this approach to higher temperature ranges where radiative losses become significant, to non-Newtonian fluids whose viscosity changes with shear rate, or to forced-convection scenarios where external stirring alters the effective cooling constant. Investigating time-varying k functions that account for the initial transient phase would also be a worthwhile extension of the present analysis.

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